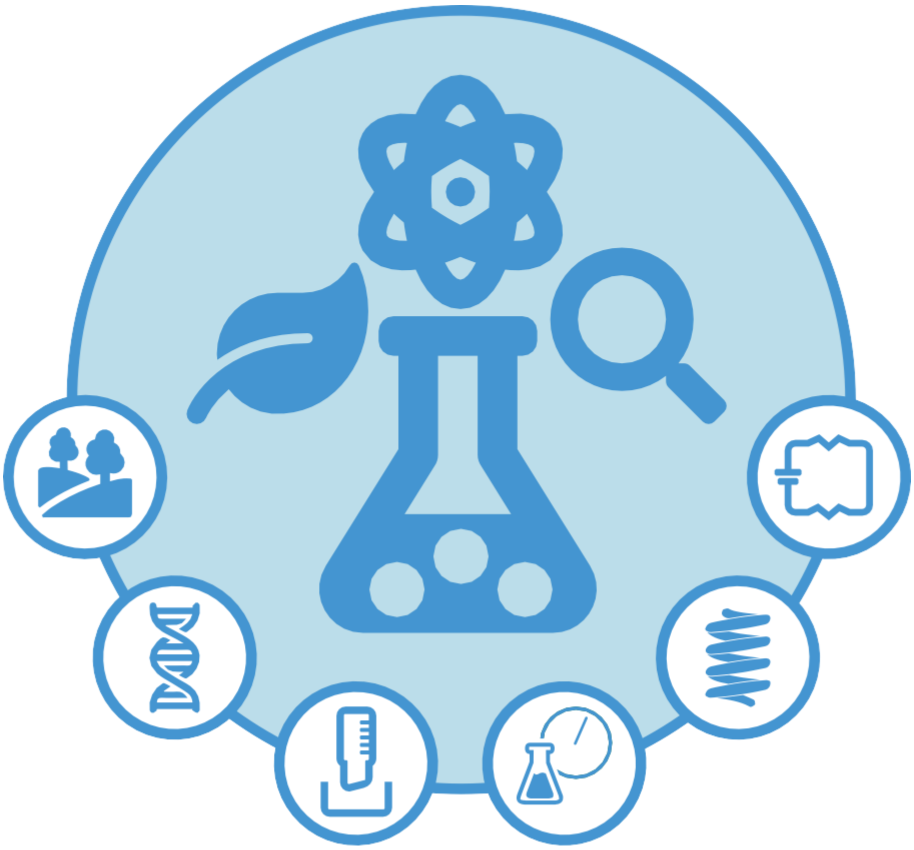
EXEMPLAR LESSON

EXPLORING THE PROPERTIES OF ELECTRONS USING A FINE BEAM TUBE



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Ministry of Education

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For information on OpenSTEM Africa see:
www.open.edu/openlearncreate/OpenSTEM_Africa

OPITO for their generous support, which has made OpenSTEM Africa and the development of the Virtual Laboratory and these materials possible.

Contents

Exemplar lessons for the OpenSTEM Africa Virtual Laboratory applications

All the **exemplar lessons** are examples of lessons which could be used both individually and by whole classes of Senior High School (SHS) students in the elective sciences of Biology, Chemistry and Physics. Each of the lessons is linked specifically to one of the applications in the OpenSTEM Africa Virtual Laboratory. The exemplar lesson is created to give, both to SHS students and to SHS teachers, a clear example of the ways in which the applications can be used in the learning and teaching of practical science. There is a focus throughout the lesson on the student's development of the practical and experimental skills which, along with knowledge and understanding, are integral to the profile of learning, teaching and assessment in SHS sciences.

The 'you' in this lesson is 'you', the Senior High School student. Remember that you can repeat the experiments and activities in this lesson as often as you have time for in class. This freedom to repeat experiments and activities is also important if you are accessing the lesson outside the classroom, for example for homework. Every application in the OpenSTEM Africa Virtual Laboratory contains real data – the experiments are real experiments. This means you might make mistakes the first or second or third time you try an experiment or an activity – and that is exactly what often happens in the real world in the sciences. So, it is helpful for you as a student to share in some of the real-world trial and error of science as you develop your skills as a scientist.

The exemplar lesson also contains a set of teaching notes at the end of this document for 'you' the SHS science teacher, to suggest how you might want to set up this particular lesson with one of your classes. Hopefully it will also generate ideas for other lessons on the same topic, or other lessons which use the same OpenSTEM Africa Virtual Laboratory application.

Investigating the properties of electrons using a fine beam tube

Lesson objectives

By the end of the lesson, you will be able to:

- Observe an electron beam in a fine beam tube (FBT)
- Change and observe the path of an electron beam
- Collect measurements from the FBT
- Use these measurements to determine the charge to mass ratio of an electron.

The following practical and experimental skills will be developed:

- Observation
- Manipulation of simulated apparatus
- Data collection
- Graph plotting
- Determining a constant from a graph.

Background

Physicists have been using charged particles in magnetic and electric fields since the end of the 19th century and will continue to do so in particle accelerators being designed for the future. Cathode ray tubes accelerating **electrons** were used in television screens and oscilloscopes until they were replaced by flat screens. In all designs for fusion reactors, magnetic forces have to be applied to keep charged plasma moving in containment vessels, then stronger fields applied in a 'magnetic pinch' to bring the particles close enough to undergo fusion reactions.

A fine beam tube (FBT) is a piece of physics laboratory apparatus designed specifically to investigate the behaviour of charged particles in magnetic fields, using an electric field to accelerate the electrons. By adjusting the voltage that accelerates the electrons, these particles can be made to travel faster or slower. By adjusting the **current** in the coils that makes magnetic field that contains the FBT, the electrons can follow circular paths of different radii. Systematic collection of measurements of the accelerating voltage, the current in the coils and radius of the beam allows you to calculate a special quantity called the charge to mass ratio of an electron.

Charged particles, such as electrons, experience a force when they are moving in a magnetic field. This force is always perpendicular to both the magnetic field and the direction of motion of the particle and can be calculated using Fleming's Left Hand Rule (which you have studied in Physics SHS2 Section 5 Unit 3).

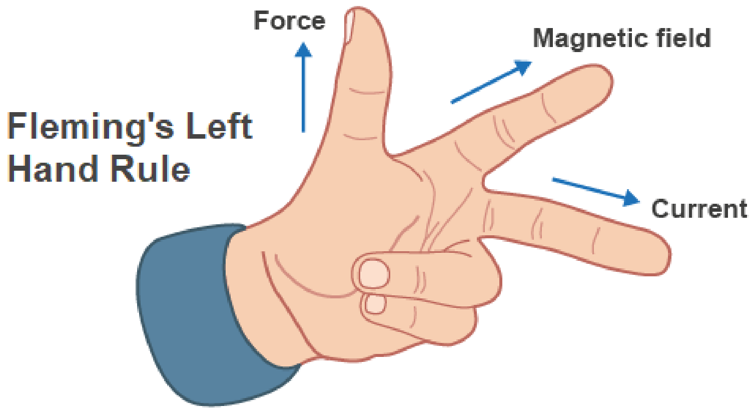


Figure 1 The use of Fleming's Left Hand Rule.

Describe the path of an electron if it experiences a magnetic force in a uniform magnetic field.
Go to Appendix 3 for the answer.

Electric fields can also be used to affect the motion of charged particles which pass through them. The big difference is in the direction – the lines of a uniform electric field are directed from positive to negative electrodes e.g., metal plates. The force created acts along the direction as the field lines. Within the FBT, the field can be used to increase the velocity of electrons from where they are released by thermionic emission at a **hot cathode**.

How can we explain that both fields in the fine beam tube accelerate electrons?

Go to Appendix 3 for the answer.

Fine beam tubes do have some features in common with a mass spectrometer. Both devices use an electric field to accelerate charged particles into a beam and both use a magnetic field to make charged particles travel in a circular path. However, the FBT does not include a velocity selector, a combination of an electric and magnetic field is used to make a mixture of different charged ions travel at a constant speed. All the electrons in the FBT travel at the same speed so that they all follow the same circular path. A mass spectrometer separates particles of different masses in the magnetic field, so they follow different paths in order to be identified.

A key feature of the FBT is that is contains a small region inside the apparatus where the electrons move in a uniform magnetic field. This is achieved using two similar coils of wire carrying a current.

The **Helmholtz configuration** is a special arrangement where two identical, current carrying coils are set up parallel to each other separated by a distance that is the same as their radius. This provides a region between them where the magnetic field is extremely close to uniform over a large enough region to do an experiment such as this, the flat section of the top curve. Note that the coils in the fine beam tube experiment are larger than those shown in this graph but the same principle applies.

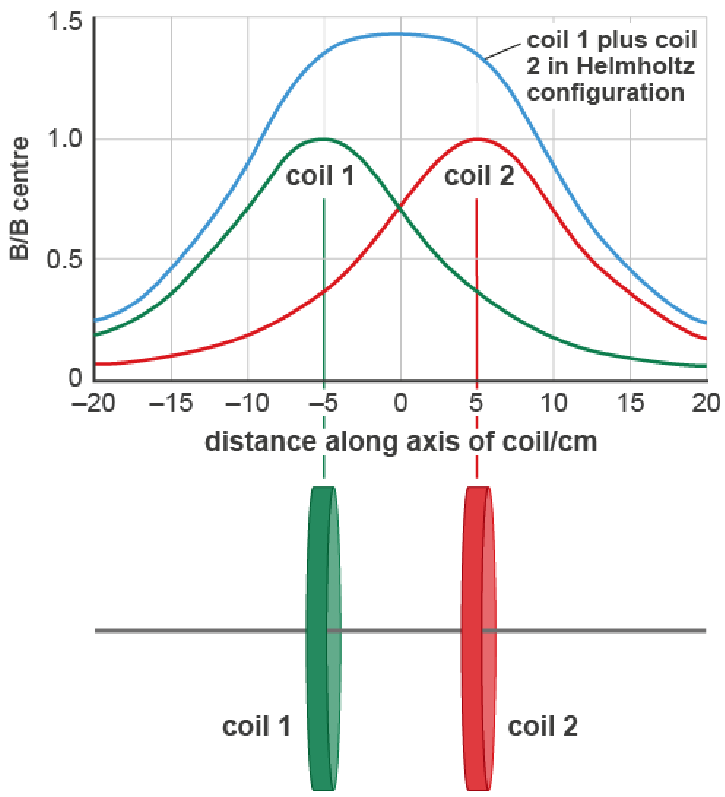


Figure 2 A graph showing the magnetic field strength along the axis of two identical coils aligned coaxially, and separated by a distance equal to their radius of 10 cm. The lower curves represent the field strength due to each coil individually, which each has a peak at

the same position as its coil. The magnetic field 5 cm from their combined centre has fallen by less than 5% compared with the value of the field midway between the coils. A combination of two coils in this configuration is known as a Helmholtz pair.

Practical activity

In this practical activity you will use a fine beam tube (FBT) to investigate the behaviour of electrons in a magnetic field. Before you get started with the practical activity the following section provides more detail about the apparatus you will be using.

Fine beam tube

Figures 3 and 4 illustrate the components that make up a FBT and these are also listed below:

- Glass container: containing very low-pressure hydrogen gas but no air. The beam is visible as photons are emitted if an electron collides with a hydrogen atom.
- Hot cathode: this is the source of the electrons used in the experiment, emitted when the cathode is heated by a small voltage – thermionic emission.
- **Anode**: a small metal cylinder vertically below the cathode.
- Beam: glows pale lilac when V is greater than $0\ V$ though can be very hard to see at low voltages.

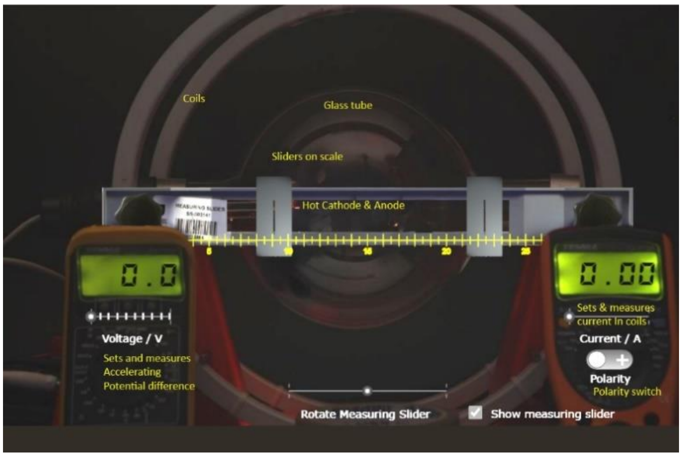


Figure 3 The fine beam tube from the application, with the beam turned off and measuring slider displayed.

External to the tube, you can see:

- Two coils in Helmholtz configuration.
- Accelerating potential/voltmeter. This potential difference (PD) is applied between the cathode and anode so that the anode is at a higher potential. This PD, V, is shown on the digital voltmeter to the left.
- Current through coils, measured on this digital ammeter.
- Polarity switch, which reverses the direction of the current through the coils.
- Measuring sliders. Use these to line up with the circular beam or coil. The distance between them when lined up = the *diameter* of the circle. These can be rotated for convenience.
- Changing V: this changes the accelerating PD in steps of approximately 20 V in the range – to 250 V.
- Changing I: this alters the current through the coils that creates the magnetic field.

Figure 4 The fine beam tube from the application in operation with a lilac circular beam shown.

Figure 5 shows a photograph of the actual FBT used in this application.

Figure 5 From left to right you can see the power supply used to accelerate the electrons, voltmeter (measures voltage), fine beam tube within the two coils, ammeter (measures current) and power supply to the coils. On the far right you can see a close-up of the lilac beam emerging downwards from the hot cathode (its location within the glass tube is indicated by the white rectangle).

Before moving onto the next section, make a sketch of and label the equipment shown in Figure 5 in your laboratory notebook.

Investigating the beam

In this part of the investigation, you will qualitatively investigate the electron beam in the FBT. Read the following instructions before accessing the FBT application.

1. Start with the accelerating PD = 0 V and increase the PD up to 250 V using the slider on the voltmeter. The polarity switch should be to the left, in the + position. Describe and explain the beam.
2. Now increase the current, using the slider on the ammeter, keeping the PD at 250 V. Describe and explain what happens to the beam
3. Reduce the current and PD to zero. Reverse the polarity of the switch to the – position. Increase the PD to 250 V then current and describe how the beam changes

Expected observations:

1. You should see the lilac electron beam pointing vertically downwards. It is brighter at higher PDs. The electrons are accelerated for a short distance through the anode and continues downward without any magnetic forces acting on it, so the electrons follow a straight path.
2. The current causes the beam to curve to the right. At low currents, the beam forms a circular arc but the magnetic is not strong enough to pull the beam into a circle. As the current increases, the force becomes strong enough to pull the beam into a circle, which gets smaller as the current/force increases.

As the beam is deflected to the right of the screen and the electrons are moving vertically downwards, what is the direction of the magnetic field?
Go to Appendix 3 for the answer.

3. The beam is now deflected in the opposite direction and leaves the magnetic field. The current in the coils has reversed, which in turn reverses the direction of the magnetic field and the force on each electron.

Explain why the system only allows you to reverse the direction of the current when it has been reduced to zero?
Go to Appendix 3 for the answer.

Making measurements with a circular beam in the FBT

In this part of the experiment you will take quantitative measurements in order to produce a graph of the results and calculate the charge to mass ratio, $\frac{e}{m_e}$. To make this clear, we will just call this number *ratio.*

In this experiment, there are three variables that can be changed and measured, current, voltage and beam radius. To make this a fair test, these are instructions to change the voltage, measure the beam radius whilst holding the current constant. This means that you are effectively seeing the change in the radius/forces on the electrons for electrons with a range of energies/velocities. The magnetic field between the coils is held constant.

1. In your laboratory notebook, copy the table below:

   Current I =

Voltage / V	Left position /cm	Right position / cm	Beam Diameter / cm	Beam Radius / m	Radius^2 / m^2

2. Turn on the apparatus and adjust the voltage to about 130 V, current to about 1.5 A so that you can see the electron beam circle. Write down the value of the current with your table and the voltage in your table.
3. Turn the slider display on and move the left-hand slider with its right edge on the left side of the circle. You should be able to leave it there for the rest of the experiment. Record slider position in the second column. The markers on the slider scale are 0.5 cm apart, try to measure the positions to the nearest mm.
4. Move the other slider so the circle is on its left-hand edge. Read and record that position in the third column.
5. Work out the diameter by subtracting the value in the second column from the third.
6. Now you need to work out the beam radius. Divide the diameter by 2 then convert the answer to meters (divide it by 100).
7. For the final column, work out the radius squared in m^2

By step 7, your data should look like this, with your own values. Note, the data in the table below is just an example.

Current I = 1.52 A

Voltage / V	Left position /cm	Right position / cm	Diameter / cm	Radius / m	Radius^2 / m^2
130.0	10.3	16.0	6.7	0.0285	0.0081

Now repeat the measurements at higher values of the voltage. You can go up in steps of 20 V up to 250 V. You will move the right-hand slider each time to measure the positions.

Graph of the data

Now use your data to plot a graph. If you plot voltage against radius on a graph, you will get a curve. In fact, you can look for a clearer relationship.

Plot voltage on the vertical (y) axis against radius^2 on the horizontal (x). You should be able to add a straight line of best fit.

Describe your graph. How would you describe the relationship shown?
Go to Appendix 3 for the answer.

Calculating a value of the ratio from one row of data

To work out the ratio, you will use the following equation, remember that $ratio = \frac{e}{m_e}$, the charge to mass ratio for an electron. B is the magnetic field and r is the radius of the beam.

$$V = \frac{1}{2}B^2r^2ratio$$

You now need to calculate B. The formula is

$$B = kI$$

k is called the *coils constant.* For the coils here, $k = 7.793 \times 10^{-4}$ T m^{-1}.

Rearrange the formula to give an expression for the ratio
Go to Appendix 3 for the answer.

Now, go to the row with your largest value of V. Use the values of V and r^2 from the table and the value you have calculated for B, to calculate the ratio.

Calculate the established value of the ratio with $e = 1.602 \times 10^{-19}$ C and $m_e = 9.109 \times 10^{-31}$ kg.

How close is your measured value to this?

Calculating a value for the ratio using the graph

Now that you have a complete table of results, you can plot a graph to work out the charge to mass ratio. This should give a more representative value that uses all of the measurements and reduces the uncertainty.

From the formula

$$V = \frac{1}{2}B^2r^2ratio$$

You can write in a form $y = mx + c$ though c should be zero here, with V on the y axis and r^2 on the x axis.

$$V = \frac{1}{2}B^2ratio \times r^2$$

You should be able to see that V is proportional to r^2 so a graph of V against r^2 gives a straight line graph through the origin. The gradient will be the expression that multiplies r^2 i.e.

$$gradient = \frac{1}{2}B^2ratio$$

How can you use the gradient to calculate the ratio?
Go to Appendix 3 for the answer.

B is the value that you have already calculated for the magnetic field. Calculate the gradient of your graph and use it to calculate the ratio.

As before, compare your value to the established value of the ratio $\frac{e}{m_e}$, using $e = 1.602 \times 10^{-19}\ \mathrm{C}$ and $m_e = 9.109 \times 10^{-31}\ \mathrm{kg}$.

Now that you have read the instructions you are ready to enter the experiment.

Fine beam tube
Go to the OpenSTEM Africa Virtual Laboratory. Click on the icon to access the Fine beam tube application homepage. Watch the introductory video before entering the experiment.

Summary

You have investigated the motion of electrons in a uniform magnetic field. You will have seen how we use the Helmholtz configuration to create a region of a uniform magnetic field. You will have observed them travelling in a circular path and used measurements to determine the charge to mass ratio of an electron which hopefully is reasonably close to the established value, using a single set of data and a graphical method.

Quiz

Answer the questions, then search for the answers in Appendix 4.

Question 1

Which kind of field makes the electrons follow a curved path?

a) Magnetic field
b) Electric field
c) Gravitational field
d) Magnetic field and electric field combined

Question 2

Which kind of field makes the electrons travel faster?

a) Magnetic field
b) Electric field
c) Gravitational field
d) Magnetic field and electric field combined

Question 3

Why would the experiment not work properly with a single coil to provide a magnetic field?

a) It would make the electrons move in the wrong direction
b) It would not provide a constant magnetic field
c) The magnetic field would be too weak for the experiment to work
d) Single coils don't affect negative particles

Question 5

For each of the statements below, select if they are true or false.

1. The direction of the magnetic force on a moving particle can be worked out with the Fleming Right Hand
2. The kinetic energy gained by an accelerating electron is equal to the potential difference x the charge on the electron
3. The Uniform field from the coils fills the whole volume of the FTB
4. One way to reduce the circular path of the electrons is to use a larger current in the coils

Glossary

Anode – Positively charged part inside the tube which attracts electrons towards it, accelerating them

Centripetal force – This is the term given to a resultant force that causes objects to move in a circle. The force acts at right angles to the object, inwards towards the centre of the circular path. In this experiment, it is due to the force on a moving charged particle (electron) in a magnetic field. It is given by the formula $F_{centripetal} = \frac{mv^2}{r}$ where m is the mass, v is the velocity of the object, r is the radius of the path.

Current – Flow of electrons in a conductor. The units are amps (A)

Electron – An electron is a particle with a charge of -e, where e = 1.602×10^{-19} C and it's mass is $m_e = 9.109 \times 10^{-31}$ kg

Fine beam Tube Glass – Vessel designed specifically to observer the motion of electrons in a uniform magnetic field and to determine the charge to mass ration of an electron.

Helmholtz configuration – Arrangement of two similar coils of wire carrying a current, which make a small region with a uniform magnetic field between them. The field has the same size and direction (at right angles to the coils) at all points within this small region

Hot cathode – A small coil that is heated to release electrons by thermionic emission

Magnetic field – Region where magnetic effects are experienced e.g. attraction to magnetic poles. Here, magnetic fields are created by currents in coils. The units are tesla (T),

Mass spectrometer – a device that measures the charge to mass ration of ions put into it. It is used typically to measure the mass of whole or partial molecules which can in turn be used to identify them. It uses electric and magnetic fields to achieve this.

Potential difference/voltage – electrical energy per unit charge. The units are volts (V).

Thermionic emission – the process where electrons are released from a metal by heating it.

Appendix 1: Teacher notes – organisation of the lesson

This lesson, using the Fine beam tube application links directly to a number of topics taught in SHS and the teaching and learning activities associated with it.

Ideas for organising this exemplar lesson link directly to activities and teaching examples in the OpenSTEM Africa CPD units on *Using ICT to Support Learning* and *Effective Questioning*.

Overview

If possible, this lesson should take place in the ICT Lab in your school if this can be arranged through your Head of Science and the Head of ICT. If the lesson takes place in the ICT Lab, it may be possible for each student to work individually at a computer; otherwise divide the class so that students are in small groups at a computer.

If it is not possible to use the ICT Lab for this lesson, then try to set up this lesson in your classroom. You may be lucky enough in your school to have a set of 'empty' tablets or mobile phones which students can use. Or you may be able to bring into the classroom a laptop connected to the internet or to your school intranet – and perhaps connected to a projector to make it possible for the whole class to view at once. If access to ICT is a real challenge in your school but you want your students to view an experiment, you might be able demonstrate it to small groups of your students at a time, using your own mobile phone

Whatever way(s) you set up the class, it would still be helpful to the students to be able to work in pairs or small groups for at least some of the lesson. Do remember as well that students need desk space to be able to write in their notebooks and to draw diagrams.

Steps in Organising the lesson

Step 1: This takes place in the lesson before the one where you and your class access the OpenSTEM Africa Virtual Laboratory fine beam tube application. Have students work in pairs to pre-read the Background section of the exemplar lesson. They should ask each other the questions in the Background section and check with each other that each understands the answers. While they are doing so, you may want to walk round the class and listen for effective questioning – and ask open and effective questions of your own – as this is important for this exemplar lesson.

Step 2: At the beginning of this exemplar lesson, check understanding by asking (again!) the students the questions in the Background section. Organise the class, if possible, to work in the same pairs as in the previous lesson. Have each person in the pair create a table for their calculations for the circular beam measurement in their own laboratory notebook in preparation for their data collection from the practical activity

Step 3: Within each pair, have them check each other's work and that each has set the table out correctly with the correct headings

Step 4: Make sure that each pair has access to a scientific calculator so that they can make the calculations. It may be that calculators will need to be shared across the class, with each pair using the calculator in turn for one set of calculations and passing it on.

Step 5: Make sure that each pair has access to/can see the computer screen to begin the actual investigation and observation and carry out the measurements. Ensure that each pair knows how to carry out the measurements – or if you are using a laptop/projector, that you draw on the expertise of the class as you go through each step of the investigation, observation and measurements – i.e., ask them what the next step is. The lilac circle showing the electron path is quite faint, and students may find it easier to do the measurements in a shaded/darkened room, or with a piece of dark fabric over them and the screen.

Step 6: Have the class follow the instructions. Make sure, if working in a pair on a PC, that each student in the pair gets to follow all the steps; if working in a group on a PC, have the group leader ensure that everyone in the group is involved.

Step 7: What they calculate and write in their copy of the table will be agreed between the pair or within the group but allow enough time for everyone in the class to fill in their own table. Have them check each other's entries with their own.

Step 8: Fifteen minutes before the end of the lesson, tell the students (in pairs) to complete the quiz. The quiz is a particularly important section of this exemplar lesson so do ensure that the students have enough time to complete it.

Note, it is likely that the data collection will take up to one hour and the analysis a further hour.

Appendix 2: Teacher notes – outputs from the lesson

The fine beam tube is a challenging experiment that draws on a number of different concepts delivered in SHS Physics Year 2. It would be worth using it only after teaching all of the material listed below. Alternatively, it could be delivered in Year 3 as a vehicle for revising these concepts.

Section	Unit	Objectives	Content
2 Mechanics	1 Energy	2.1.1, 2.1.2	Energy and conservation of energy
	2 Circular motion	2.2.2	Centripetal force
5 Electricity and magnetism	2 Magnets	5.2.2	Magnetic field
	3 Electro-magnetism	5.3.1	Magnetic fields created by current
		5.3.2	Fleming's Left- Hand Rule
		5.3.6	Force on a charged particle in a magnetic field and Electric field
6 Atomic and nuclear physics	2 Thermionic Emission	6.2.1, 2	Thermionic emission and cathode rays

Comments on *Background*

The fine beam tube (FBT) is an application that uses thermionic emission to produce an electron beam (Unit 2 Objectives 6.2.1, 2). The electric field forces in the anode accelerate the electrons and their speed can be calculated by conservation of energy (Unit 1, 2.1.1,2) such that the electrical work done accelerating the electrons = gain in kinetic energy.

The two coils produce a magnetic field between them (Unit 3, 5.2.2, 5.3.1). The field produces a force on the electrons, the direction is given by Fleming's Left Hand Rule (Unit 3, 5.3.2) and the magnitude of the force using the formula given in Unit 3, section 5.3.6. Since the direction of the force is perpendicular to the velocity of the electrons, the force provides a centripetal acceleration (Unit 2, section 2.2.2) making the electrons travel in a circle.

However, the Helmholtz configuration is not introduced in SHS Physics, so this is an extension exercise, which serves as a synoptic task. By combining the equations given in these sections with additional equations for the specific magnetic field here, it is possible to derive a framework to investigate the relationship between

- The voltage used to accelerate the electrons (linked to their speed), *V*
- The current in the coils (which causes the magnetic force), *I*
- The radius of the electron beam, *r*

The exemplar shows how one relationship can be investigated graphically. At a higher level, the data can be used to determine the charge to mass ratio of an electron $\frac{e}{m_e}$, using a single set of data or the gradient of a graph

Comments on *Practical Activity*

This section starts with a detailed account of the apparatus and its parts. The experiment is an *Interactive Screen Experiment* which is based on many photographs of a real set of apparatus showing all possible combinations of current, voltage, polarity and the resulting beam. Changing a slider changes the image smoothly to a different image with a different set of parameters so is close to running the experiment for real.

Comments on *Investigating the Beam*

This is a series of three short tasks to qualitatively investigate the behaviour of the beam, changing the voltage, current and polarity. For some student groups, it may only be sufficient to explore this task.

Comments on *Making Measurements with a circular beam in the FBT*

This is the quantitative investigation. It explains the data collection, graph plotting and calculation of $\frac{e}{m_e}$.

Using just data and graphically. These tasks build in difficulty so that the teacher can decide which tasks are appropriate to their students.

The algebra may be on the limit of what students are prepared for and has been simplified as much as possible to calculate $\frac{e}{m_e}$. Students will need access to a scientific calculator (e.g., one on a computer) as they will manipulate small numbers written in scientific notation and plot small numbers on the graph.

Mathematical background

Symbols used:

V = accelerating voltage

q = charge on a particle

e = charge on an electron = 1.602×10^{-19} C

m = mass of a particle

m_e = electron mass = 9.109×10^{-31} kg

v = velocity of an electron

B = magnetic field

F = force

r = radius of circular beam

θ = angle between magnetic field and velocity, 90° in this context

ratio = e/m_e = 1.76×10^{11} C kg^{-1}

N = number of turns of wire in each of the coils = 130 here

R = radius of each coil = 15 cm here

μ_0 = permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

k = coils constant = 7.793×10^{-4} T m^{-1} for these coils

Work done on electron = gain in kinetic energy.

Work done accelerating through $V = qV, q = e$ and mass = m_e

$$eV = \frac{1}{2}m_e\, v^2 \quad (1)$$

The magnetic field force $F = B\,q\,v \sin\theta$ where $q = e$ and $\theta = 90°$

The magnetic field provides the centripetal force, $F = \frac{mv^2}{r}$

$$Bev = \frac{m_e v^2}{r} \quad (2)$$

From (2)

$$Ber = m_e v$$
$$v = \frac{Ber}{m_e}$$
$$v^2 = \left(\frac{Ber}{m_e}\right)^2$$

Using (1)

$$eV = \frac{1}{2}m_e\left(\frac{Ber}{m_e}\right)^2$$
$$V = \frac{1}{2}B^2r^2\left(\frac{e}{m_e}\right)$$

In the language used here, where $ratio = \frac{e}{m_e}$

$$V = \frac{1}{2}B^2r^2\, ratio \quad (3)$$

The last step is the relationship between B and I. The theory here is well beyond the level of SHS, so here is the given formula.

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 NI}{R}$$

This is simplified to $B = kI$ where

$$k = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 N}{R}$$

For this coil, R = 15.0 cm = 0.0150 m and N = 130 terms. Hence the coils constant k = 7.793×10^{-4} T m^{-1} for this apparatus

The final formula that links the experiment

$$V = \frac{1}{2} k^2 I^2 r^2 ratio$$

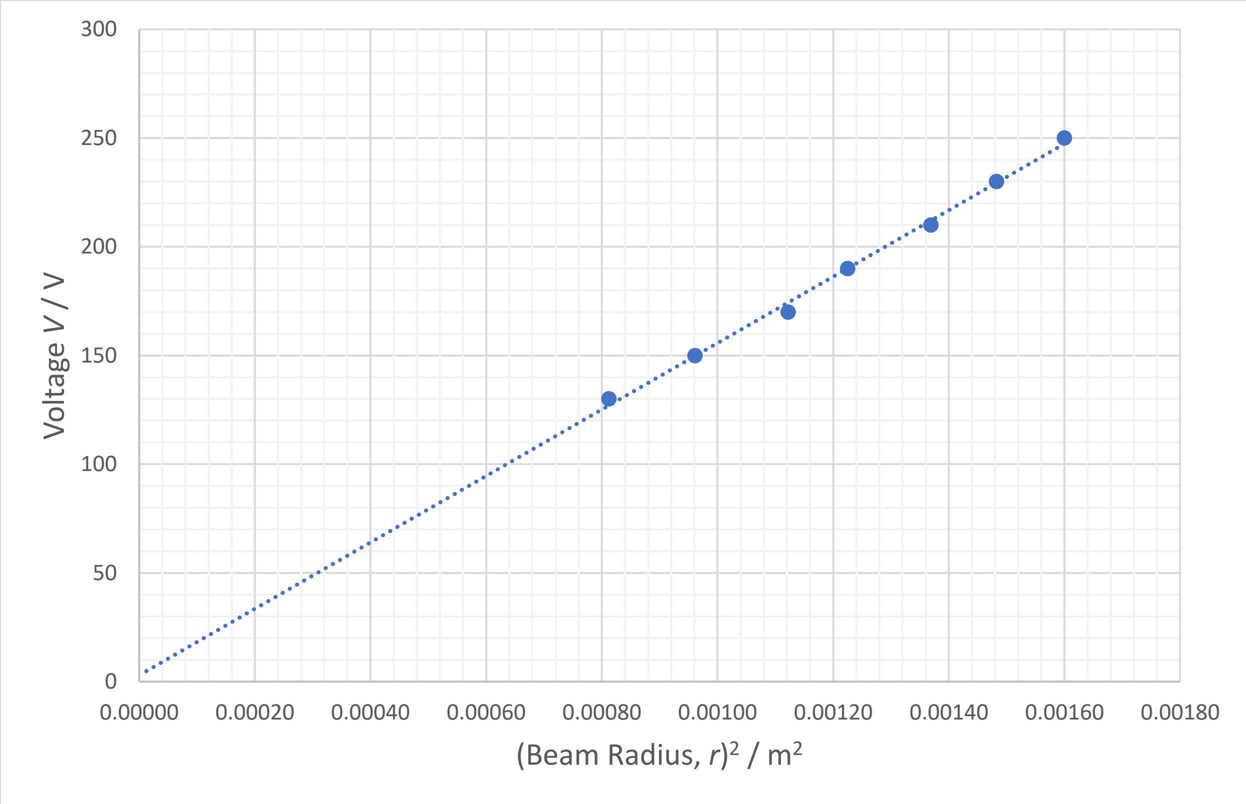
This can be used as the basis of any investigation where you might want to fix one of (*V, I, r*) and vary/measure the other two.

Sample dataset

The experiment has three variables – *V, I* and *r*. It is possible to choose a constant value of one and investigate the relationship of the other two. Here, the instructions investigate the relationship between *V* and *r* ($V \propto r^2$) with a fixed current as it is the easiest process to explain. It is possible to do two more investigations fixing *r* or *V*. The data can be manipulated into a straight-line graph for either of these combinations and the gradient used to determine $\frac{e}{m_e}$. This could also be a planning exercise for some students.

Current I = 1.52 A

Voltage / V	Left position /cm	Right position / cm	Diameter / cm	Radius / m	Radius^2 / m^2
130	10.3	16.0	5.7	0.0285	0.00081
150	10.3	16.5	6.2	0.0310	0.00096
170	10.3	17.0	6.7	0.0335	0.00112
190	10.3	17.3	7.0	0.0350	0.00123
210	10.3	17.7	7.4	0.0370	0.00137
230	10.3	18.0	7.7	0.0385	0.00148
250	10.3	18.3	8.0	0.0400	0.00160



Graph showing Voltage against (Beam Radius)^2 for the Fine Beam Tube, current fixed at 1.52 A

The gradient is 1.53×10^5 V m^{-2}

Sample single calculation of $\frac{e}{m_e}$

Using the last row of the table and the answer to ITQ 6

$$B = k\,I$$
$$= \ 7.793 \times 10^{-4}\ T\ m^{-1} \times 1.52\ A$$
$$= 0.0011845\ T$$

$$ratio = \frac{2V}{B^2 r^2}$$
$$= \frac{2 \times 250\ V}{(0.0011845\ T \times .0400\ m)^2}$$
$$= 2.23 \times 10^{11}\ C\ kg^{-1}$$

Sample calculation $\frac{e}{m_e}$ from graph

Using the answer to ITQ 7, *B* calculated above and the gradient of the graph

$$ratio = \frac{2 \times 1.53 \times 10^5\ V\ m^{-2}}{(0.0011845\ T)^2}$$
$$= 2.18 \times 10^{11} C\ kg^{-1}$$

Final value of $\frac{e}{m_e}$ and uncertainties

The calculated value of e/m_e in this experiment is consistently higher than the established value of 1.76 × 10^{11} C kg^{-1}. With care, it is possible to get a graph showing a good straight line of best fit, through the origin, with little scatter around the line. There is plenty of scope for considering the uncertainties in the measurements of beam radius, voltage and current. There is scope to instruct students to repeat measurements of the radius.

The diameter of the coil and number of turns in each coil uses values provided by the manufacturer. They in turn are used in the calculation of k used to determine the value of the magnetic field. That assumes that the separation of the 2 coils and alignment is as close as possible to the ideal Helmholtz configuration.

Extension questions

Below are some extension questions you may wish to use with your more advanced students.

Question A

Why does an increase in the coil current lead to a smaller radius for the electron beam?

Answer for Question A

The magnetic field is proportional to this current so it increases. The force provides a stronger centripetal force, $F_{centripetal} = \frac{mv^2}{r}$ which in turn is inversely proportional to the radius of the path. So, a stronger field supports a smaller circle for constant velocity.

Question B

Why would you not be able to perform this experiment with a vacuum in the vacuum tube?

Answer for Question B

You would not be able to see the electron beam. The faint lilac glow of the beam arises from collisions between the electrons and hydrogen atoms. The electron within the atom is excited to a higher energy level

Question C

Why did we ask you to do experiments with quite small circles, radius up to about 4 cm?

Answer for Question C

The Helhmoltz configuration of two coils is about as good as a physicist can do to produce a uniform magnetic field. It is extremely close to uniform for a cylindrical region between these coils up to a radius up to about 4 cm from the axial line through the centre of the 2 coils. Thereafter, it decreases significantly. This experiment is constrained by having the beam within the region which shows the most uniformity. So, we asked you to use a current of about 1.5 A to limit this

You could use a bigger current and smaller circles, but that may not give the best results. The beam can only be measured with limited precision (e.g. random uncertainty of ± 1 – 2 mm, so a smaller radius would have a larger percentage of fractional uncertainty).

More challenging questions

Question D

Calculate the velocity of the electron using your value of $\frac{e}{m_e}$ when the accelerating voltage = 220 V.

Hint – the electrical energy given to the electron is its charge × voltage and this is converted into kinetic energy $KE = \frac{1}{2}mv^2$

Answer for Question D

Using $eV = \frac{1}{2}mv^2$

Rearrange for v, $v = \sqrt{2\frac{e}{m_e}V}$

Example, with $V = 220\text{ V}$ and $\frac{e}{m_e} = 1.8 \times 10^{11}\text{ C kg}^{-1}$

$$v = \sqrt{2 \times 1.8 \times 10^{11}\text{ C kg}^{-1} \times 220\text{ V}}$$
$$v = 8.9 \times 10^{6}\text{ m s}^{-1}$$

So, the voltage accelerates the electron to almost 10 million m s^{-1}.

Question E

Calculate $\frac{e}{m_e}$ with the following data – V = 220 V, I =1.60 A, r_{beam}= 0.040 m

Answer to Question E

Using

$$V = \frac{1}{2}B^2r^2 ratio \quad (1)$$

First work out B

$$B = kI$$
$$= 7.793 \times 10^{-4}\, T\, m^{-1} \times 1.60\, A$$
$$= \; 0.0012467\, T$$

Rearranging (1) and using the value for *B*

$$ratio = \frac{2V}{B^2r^2}$$

$$= \frac{2 \times 220\ V}{(0.0012467\ T)^2 \times (0.040\ m)^2}$$

$$\frac{e}{m_e} = 1.8 \times 10^{11} \text{C kg}^{-1}$$

Appendix 3: In-text question answers

Describe the path of an electron if it experiences a magnetic force in a uniform magnetic field.
Answer: The path will be circular. The force $\boldsymbol{F_{mag}}$ acts at right angles to the path of the electron. This provides a **centripetal force** to maintain circular motion. A uniform magnetic field has the same magnitude and direction at each point in the field, which can be represented with parallel straight magnetic field lines. The electron will maintain a circular path for as long as it remains in the region with a uniform field.

How can we explain that both fields in the fine beam tube accelerate electrons?
Answer: Electric fields are used to change the speed of the electrons without altering their direction. Magnetic forces act perpendicular to the path of the electrons so alter the direction of the electrons without altering the speeds. These are both forms of acceleration as they change the velocity, which is a vector quantity with speed and direction.

As the beam is deflected to the right of the screen and the electrons are moving vertically downwards, what is the direction of the magnetic field?
Answer: Using the Fleming Left Hand Rule, the conventional current I is vertically upwards so the magnetic field is directed outwards from the plane of the coils towards the experimenter.

Explain why the system only allows you to reverse the direction of the current when it has been reduced to zero?
Answer: This is a safety feature. In practice, swapping the polarity means pulling out and swapping the connectors for 2 leads. It is safer to do this when the apparatus is switched off.

Describe your graph. How would you describe the relationship shown?
Answer: Your graph should show a good straight-line relationship, with points very close to the line of best fit. The line of best fit should pass through or very close to the origin, meaning that the relationship here is *voltage is proportional to radius*^{2}.

Rearrange the formula to give an expression for the ratio

Answer:

$$ratio = \frac{2V}{B^2 r^2}$$

To calculate with this, you could use this version:

$$ratio = 2 \times V \div (B^2 \times r^2)$$

How can you use the gradient to calculate the ratio?

Answer:

Using the formula

$$gradient = \frac{1}{2} B^2 \, ratio$$

Rearrange this to calculate

$$ratio = \frac{2 \times gradient}{B^2}$$

Appendix 4: Quiz answers

Correct answers are **highlighted in green**.

Question 1

Which kind of field makes the electrons follow a curved path?

a) Magnetic field
b) Electric field
c) Gravitational field
d) Magnetic field and electric field combined

Question 2

Which kind of field makes the electrons travel faster?

a) Magnetic field
b) Electric field
c) Gravitational field
d) Magnetic field and electric field combined

Question 3

Why would the experiment not work properly with a single coil to provide a magnetic field?

a) It would make the electrons move in the wrong direction
b) It would not provide a constant magnetic field
c) The magnetic field would be too weak for the experiment to work
d) Single coils don't affect negative particles

Question 5

For each of the statements below, select if they are true or false.

1. The direction of the magnetic force on a moving particle can be worked out with the Fleming Right Hand
   (**False**, it's the Left Hand rule)

2. The kinetic energy gained by an accelerating electron is equal to the potential difference x the charge on the electron (**True**)

3. The Uniform field from the coils fills the whole volume of the FTB
   (**False**, it is only a region within a few cm of the middle of the tube).

4. One way to reduce the circular path of the electrons is to use a larger current in the coils
   (**True**, the magnetic field is proportional to this current).

ACKNOWLEDGEMENTS

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